Journal of Engineering Physics and Thermophysics, Vol. 68, No. 6, 1995

TEMPERATURE REGIMES CONTROL DURING THERMAL TREATMENT OF BODIES WITH ORGANIC THERMOREACTIVE COATINGS

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UDC 621.315.33:536.33.01

An analytical solution for heating (cooling) of a flat composite plate with radiation-convection heat supply and boundary conditions of the first kind is obtained and analyzed.

Composite materials with protective and protective-decorative polymer thermoreactive coatings open up prospects for widening the range of starting materials for the building industry using the mineral and organic by-products of a number of branches of industry, including military industry. Building materials with these coatings are more long-lived, due to substantial improvement of their water resistance, strength, frost resistance, and other product-quality indices [1-3].

The development of energy-saving technologies for the production of these materials involves the design of thermal devices that would provide optimal regimes for physicochemical processes in the coating and satisfy restrictions on temperature levels of the substrate to prevent heat damage. This can be realized on the basis of an understanding of the relationships between the parameters that determine the thermal regimes of heating devices and the temperature fields in coated items.

The actual technology of heat treatment of items with thermoreactive coatings implies continuousness of the process and accuracy of maintaining temperature regimes which can be realized in continuously operating electric furnaces. The maximum temperatures of the radiating surfaces of the working space of these heating devices are limited by the serviceability of construction materials utilized. For widely used and easily accessible heaters, the maximum working temperature is 1300...1400 K. In order to provide continuous removal of solvent vapors, which constitute a dangerously explosive mixture, from the working space and prevent condensation of the vapors on the surfaces of the working space, a heat-carrier circulation system is necessary. At the same time, the temperature of the heat carrier should not exceed 600 K. Thus, the temperature regime of items with thermoreactive coatings is determined by the radiative and convective components of the heat flux that arrives at the coating surfaces from the radiating surfaces of the working space and the convecting heat carrier.

We consider the problem of heating (cooling) of gypsum plates of standard dimensions ($0.15 \times 0.15 \times 0.005...0.007$ m) with an applied coating 0.2...0.4 mm thick. Considering that the thickness of the plate is more than 15 times smaller than its width and length and that the plate material has low heat conductivity, the problem can be considered one-dimensional to a good approximation. Depending on the gypsum, it can be heated up to a temperature of 330...450 K. Within this tempearture region the nonlinearity connected with the temperature dependence of thermophysical properties of the substrate and coating can also be neglected.

If the thermal effects of endo- and exothermic reactions in the thin coatings 1 (Fig. 1) can be neglected, and constancy of the thermophysical properties of the coating and the item 2 can be assumed, the most simple mathematical model that describes these relationships for a system of flat bodies is as follows:

$$\frac{\partial T_1(x,\tau)}{\partial \tau} = a_1 \frac{\partial^2 T_1(x,\tau)}{\partial x^2}, \quad \delta_2 \le x \le \delta_1 + \delta_2, \quad \tau > 0; \tag{1}$$

Scientific Research Institute of Building Materials, Tomsk, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 68, No. 6, pp. 991-997, 1995. Original article submitted December 29, 1993; revision submitted April 4, 1995.

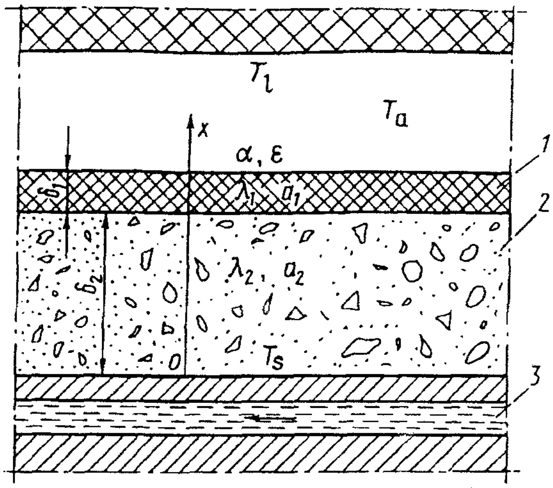

Fig. 1. Formulation of the problem: 1) coating; 2) item; 3) port.

$$\frac{\partial T_2(x, \tau)}{\partial \tau} = a_2 \frac{\partial^2 T_2(x, \tau)}{\partial x^2}, \quad \delta_2 \le x \le \delta_1 + \delta_2, \quad \tau > 0; \tag{2}$$

$$T_1(x, 0) = T_2(x, 0) = T_0; \tag{3}$$

$$\frac{\partial T_1(\delta_1 + \delta_2, \tau)}{\partial x} + \frac{\alpha}{\lambda_1}[T_a - T_1(\delta_1 + \delta_2, \tau)] + \varepsilon\sigma_0 [T_e^4 - T_1^4(\delta_1 + \delta_2, \tau)] = 0; \tag{4}$$

$$\frac{\partial T_1(\delta_2, \tau)}{\partial x} = \frac{\lambda_2}{\lambda_1}\frac{\partial T_2(\delta_2, \tau)}{\partial x}, \quad T_1(\delta_2, \tau) = T_2(\delta_2, \tau); \tag{5}$$

$$T_2(0, \tau) = T_s. \tag{6}$$

Numerical solution of the system of equations was carried out in [4-6]. Parametric analysis of the problem showed that the duration of thermotreating of coatings on plates from various materials 0.1 m thick does not exceed 10 min. At the same time, calculation of thermotreatment regimes requires more than one hour of CPU time of an IBM PC AT-compatible computer, which restricts possible applications of the software in real-time control systems of temperature regimes. This circumstance pointed to the necessity of finding an analytical solution of system of Eqs. (1)-(6).

In the case of a purely convective heat supply to the coating surface, an exact analytical solution was found by the method of the integral Laplace transforms. This solution was used to estimate the degree of accuracy of the approximate analytical solution of system of Eqs. (1)-(6), which we managed to obtain using a piecewise linear approximation of the parabola of the fourth power of the desired temperature of the coating surface:

$$T_1^4(\delta_1 + \delta_2, \tau) = a_i + b_i T_1(\delta_1 + \delta_2, \tau);$$

$$b_i = \frac{T_i^4(\delta_1 + \delta_2, \tau) - T_{i-1}^4(\delta_1 + \delta_2, \tau)}{T_i(\delta_1 + \delta_2, \tau) - T_{i-1}(\delta_1 + \delta_2, \tau)};$$

$$a_i = T_{i-1}^4(\delta_1 + \delta_2, \tau) - b_i T_i(\delta_1 + \delta_2, \tau).$$

Within the region of coating-surface temperature of from $T_i^{i-1}(\delta_1 + \delta_2, \tau)$ to $T_1^i(\delta_1 + \delta_2, \tau)$, where the coefficients a_i and b_i can be calculated beforehand, the solution takes the following form:

TABLE 1. Temperature Field (T, K) of Gypsum Items with Coating in the Vicinity of the Coating Surface in the Initial Time Instants

Distance from the coating surface, $x\cdot 10^{-3}$ m	$\tau = 10$ sec			$\tau = 30$ sec		
	$\delta_2\cdot 10^{-3}$ m					
	5.0	7.5	10.0	5.0	7.5	10.0
0	432,6	429,7	430,7	506,6	507,4	508,5
0,05	426,9	424,0	–	501,3	502,2	–
0,10	421,3	418,5	419,4	496,1	497,0	498,1
0,15	415,9	413,0	–	491,0	491,8	–
0,20	410,6	407,7	408,7	485,9	486,8	487,9

$$T_1(x,\tau) = T_0 + \frac{\varphi_3^2 (K_\lambda + X - 1) + 2T_s - T_0)\,[\beta_2 - \beta_1 (X-1)]}{(\alpha + \varepsilon\sigma_0 b_i)\,M_0} +$$

$$+ \sum_{n=1}^{\infty} \frac{(M_1 + M_2 - M_3 - M_4)\cos\mu_n X +}{N_1 + N_2 - N_3 - N_4 + N_5 + N_6 + N_7 + N_8 +} \rightarrow$$

$$\rightarrow \frac{+ (M_5 + M_6 - M_7 - M_8)\sin\mu_n X}{+ N_9 + N_{10} - (N_{11} + N_{12} + N_{13} + N_{14} + N_{15} + N_{16})} \exp(-\mu_n^2 \mathrm{Fo}_1)\,; \qquad (7)$$

$$T_2(x,\tau) = T_0 + \frac{(\alpha T_a - \alpha T_0 - \varepsilon\sigma_0 T_e^4 - \varepsilon\sigma_0 a_i - \varepsilon\sigma_0 b_i T_0)\,X}{(\alpha + \varepsilon\sigma_0 b_i)\,M_0} +$$

$$+ \sum_{n=1}^{\infty} \frac{(T_s - T_0)\cos\mu_n X + (M_9 + M_{10} + M_{11} - M_{12} -}{N_1 + N_2 - N_3 - N_4 + N_5 + N_6 + N_7 + N_8 + N_9 +} \rightarrow$$

$$\rightarrow \frac{- M_{13} - M_{14})\sin\mu_n X}{+ N_{10} - (N_{11} + N_{12} + N_{13} + N_{14} + N_{15} + N_{16})} \exp(-\mu_n^2 \mathrm{Fo}_2)\,. \qquad (8)$$

The values of the roots μ_n at $1 \le n \le \infty$ are calculated from the characteristic equation

$$\mathrm{ctan}\,\mu_n = \frac{P_1 + P_2 - P_3}{\mu_n \beta_3 K_a (P_4 + P_5)}\,. \qquad (9)$$

In expressions (7)-(9), the following notation is used:

$$M_0 = 3 - K_\lambda - 9\beta_2/\beta_1 - 7K_\lambda (1 + K_\delta)/2\,;$$

$$M_1 = (T_s - T_0)\cos\mu_n K_a\,[\beta_1 \sin\mu_n K_\delta^* - \mu_n \beta_2 \cos\mu_n K_\delta^*]\sin\mu_n K_a\,;$$

$$M_2 = \varphi_3 \sin\mu_n\,[\mu_n \beta_2 K_a \cos(\mu_n K_a)]\,; \quad M_3 = \varphi_3 \mu_n \beta_3 \sin(\mu_n K_a)\cos\mu_n\,;$$

$$M_4 = \mu_n \beta_2 K_a \cos(\mu_n K_a)\cos\mu_n\,[\beta_1 \sin(\mu_n K_\delta^*) - \mu_n \beta_3 \cos(\mu_n K_\delta^*)]\,;$$

$$M_5 = (T_s - T_0)\cos\mu_n K_a\,[\mu_n \beta_2 K_a \cos(\mu_n K_a)]\,K_7\,;$$

$$M_6 = -\varphi_3 \mu_n \beta_2 \sin^2\mu_n K_a\,; \quad M_7 = -\mu_n \beta_2 K_a (T_s - T_0) K_7 \sin^2(\mu_n K_a)\,;$$

$$M_8 = -\varphi_3 \mu_n \beta_2 K_a \cos\mu_n \cos(\mu_n K_a)\,; \quad M_9 = \varphi_3 \mu_n \beta_2 K_a \cos\mu_n \cos(\mu_n K_a)\,;$$

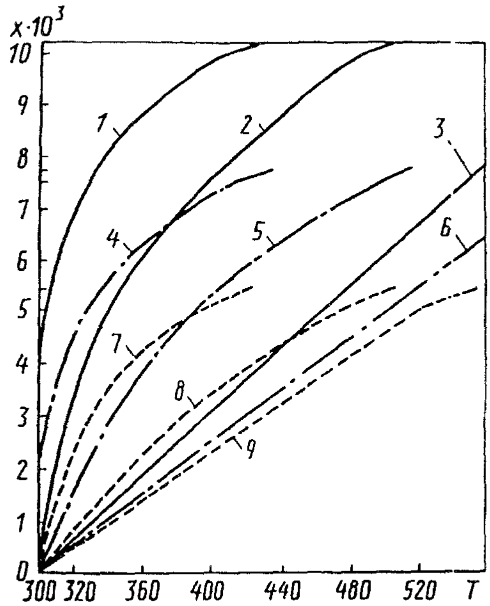

Fig. 2. Character of temperature-field variation in gypsum item and thermoreactive coating for $\tau = 10$ (1, 4, 7); 30)2, 5, 8), and 180 sec (3, 6, 9): 1-3) $\delta_1 = 0.2 \cdot 10^{-3}$ m, $\delta_2 = 10 \cdot 10^{-3}$ m; 4-6) $\delta_1 = 0.2 \cdot 10^{-3}$ m, $\delta_2 = 7.5 \cdot 10^{-3}$ m; 7-9) $\delta_1 = 0.4 \cdot 10^{-3}$ m, $\delta_2 = 5.0 \cdot 10^{-3}$ m. x, thickness coordinate, m; T, temperature, K.

$$M_{10} = -\ [(T_s - T_0)\,\mu_n\beta_3 \sin(\mu_n K_a) \cos(\mu_n K_a)]\,K_8\,; \quad M_{11} = [\mu_n\beta_2 K_a (T_s - T_0) \sin\mu_n \sin(\mu_n K_a)]\,K_7\,;$$

$$M_{12} = -\ [\mu_n\beta_3 (T_s - T_0) \cos(\mu_n K_a) \cos\mu_n]\,K_7\,; \quad M_{13} = \varphi_3 \sin\mu_n\,[\mu_n\beta_3 \sin\mu_n K_a]\,;$$

$$M_{14} = \mu_n K_a \beta_2 (T_s - T_0)\,[\cos\mu_n \sin\mu_n K_a]\,K_8\,; \quad N_1 = \mu_n\beta_2 K_a\,[\sin\mu_n \cos\mu_n K_a]\,K_7\,;$$

$$N_2 = \mu_n\beta_3\,[\sin^2(\mu_n K_a)]\,K_8\,; \quad N_3 = \mu_n\beta_3\,[\cos\mu_n \sin\mu_n K_a]\,K_7\,;$$

$$N_4 = \mu_n\beta_2 K_a\,[\cos\mu_n \cos\mu_n K_a]\,K_8\,; \quad N_5 = 0.5\mu_n^2\beta_2 K_a\,[\cos\mu_n \cos\mu_n K_a]\,K_7\,;$$

$$N_6 = -\ \mu_n^2 K_a^2 \beta_2\,[\sin\mu_n \sin(\mu_n K_a)]\,K_7\,;$$

$$N_7 = 0.5\mu_n^2\beta_2 K_a\,[\sin\mu_n \cos\mu_n K_a]\left\{\mu_n\beta_3 K_\delta^* \cos(\mu_n K_\delta^*) - [\beta_3 + \beta_1 K_\delta^* \sin(\mu_n K_\delta^*)]\right\};$$

$$N_8 = -\ 0.5\mu_n^2\beta_3 K_a\,[\cos(\mu_n K_a) \sin(\mu_n K_a)]\,K_8\,; \quad N_9 = -\ 0.5\mu_n\beta_3 \sin(\mu_n K_a)\,[\mu_n \cos\mu_n + \sin\mu_n]\,K_\delta\,;$$

$$N_{10} = 0.5\mu_n^2\beta_3 \sin^2(\mu_n K_a)\,[(\beta_3 - \beta_1 K_\delta^* \cos(\mu_n K_\delta^*) - \mu_n\beta_3 K_\delta^* \sin(\mu_n K_\delta^*)]\,;$$

$$N_{11} = -\ 0.5\mu_n^2 K_a \cos(\mu_n K_a)\,(\beta_3 \cos\mu_n)\,K_7\,; \quad N_{12} = 0.5\mu_n\beta_3 \sin(\mu_n K_a)\,(\cos\mu_n - \mu_n \sin\mu_n)\,K_7\,;$$

$$N_{13} = 0.5\mu_n^2\,[\beta_3 \cos\mu_n \sin(\mu_n K_a)]\,[\mu_n\beta_3 K_\delta^* \cos(\mu_n K_\delta^*) + (\beta_3 - \beta_1)\,K_\delta^* \sin(\mu_n K_\delta^*)]\,;$$

$$N_{14} = 0.5\mu_n^2\beta_3\,[\sin\mu_n \cos(\mu_n K_a)]\,K_8\,; \quad N_{15} = 0.5\mu_n\beta_3\,[\cos\mu_n \cos(\mu_n K_\delta^*)]\,K_8\,;$$

$$N_{16} = 0.5\mu_n^2\beta_3\,[\cos\mu_n \cos(\mu_n K_a)]\,[\mu_n\beta_3 K_\delta^* \sin(\mu_n K_\delta^*) - (\beta_3 - \beta_1)\,K_\delta^* \cos(\mu_n K_\delta^*)]\,;$$

$$P_1 = \mu_n\beta_3 \sin(\mu_n K_a) \cos\mu_n \sin^{-1}\mu_n\,(\mu_n\beta_3 \sin\mu_n K_\delta^* + \beta_1 \cos\mu_n K_\delta^*)\,;$$

$$P_2 = \mu_n\beta_3 \sin(\mu_n K_a) \sin^{-1}\mu_n\, [\beta_1 \sin(\mu_n K_a) \sin(\mu_n K_\delta) - \mu_n\beta_3 \sin(\mu_n K_a) \cos(\mu_n K_\delta)]\,;$$

$$P_3 = -\mu_n\beta_2 K_a \cos(\mu_n K_a)\, [\mu_n\beta_3 \sin(\mu_n K_\delta^*) + \beta_1 \cos(\mu_n K_\delta^*)]\,;$$

$$P_4 = \mu_n\beta_3 \cos(\mu_n K_a) \sin(\mu_n K_\delta^*)\,; \quad P_5 = \beta_1 \cos(\mu_n K_a) \sin(\mu_n K_\delta^*)\,;$$

$$K_7 = \mu_n\beta_3 \sin(\mu_n K_\delta^*) + \beta_1 \cos(\mu_n K_\delta^*)\,; \quad K_8 = \beta_1 \sin(\mu_n K_\delta^*) - \mu_n\beta_3 \cos(\mu_n K_\delta^*)\,;$$

$$\varphi_1 = (T_s - T_0) \cos(\mu_n K_a)\,; \quad \varphi_2 = -\mu_n\beta_2 K_a \sin(\mu_n K_a)\,;$$

$$\varphi_3 = \alpha T_a - \alpha T_0 - \varepsilon\, \sigma_0 T_e^4 - \varepsilon\, \sigma_0 a_i - \varepsilon\, \sigma_0 b_i T_0\,;$$

$$\beta_1 = \alpha + \varepsilon\sigma_0 b_i\,; \quad \beta_2 = \lambda_2/\delta_2\,; \quad \beta_3 = \lambda_1/\delta_1\,; \quad X = x/\delta_2\,;$$

$$K_a = \sqrt{a_1/a_2}\,; \quad K_\delta = \delta_1/\delta_2\,; \quad K_\delta^* = 1 + K_\delta\,; \quad \mathrm{Fo}_1 = a_1\tau/\delta_1^2\,; \quad \mathrm{Fo}_2 = a_2\tau/\delta_2^2\,.$$

When the analytical expressions obtained are used, the CPU time for calculations of the temperature regime of thermal treatment of coated gypsum plates is less than 50 sec. This allows us to recommend dependences (7) and (8), despite their awkwardness, for a designer's thermal calculations of corresponding furnaces and for automated temperature control systems in furnaces.

Application of approximate analytical solutions (7) and (8) also made it possible to reveal the influence of the main parameters that determine the intensity of external heat transfer on the process of temperature-field formation in a composite flat body.

Table 1 presents calculation results for the temperature fields in the vicinity of the coating surface in the initial instants of thermal treatment. It is evident that the temperature of the item's surface at the level at which restrictions are imposed due to thermal destruction phenomena is virtually equal at the same instants, irrespectively of the thickness of the item. This important conclusion makes it possible to perform thermal treatment in the fast heating zone at the same regime parameters of the heater, irrespectively of the item thickness.

Figure 2 shows the character of variation of the temperature fields in a composite flat body at different instants up to those corresponding to conditions close to those of stationary heat transfer. The calculations were carried out for various thicknesses of gypsum plates and coatings based on PI-25 thermoreactive resin.

The optimum degree of completeness of the reaction of chemical structuring of the coating [8] is within the limits of 0.8...0.85 and can be determined from the expression

$$A = 1 - \exp \int_0^\tau - k_0 \exp\left(\frac{U}{RT_1(\tau)}\right) d\tau\,. \tag{10}$$

The problem of control of the temperature regimes of the item and coating involves the choice and maintenance of parameters that determine external heat transfer for the restrictions imposed on the maximum temperature of the item $T_2(\delta_2, \tau)$ and mean-integral coating temperature such that the quantity A is within the limits of 0.8...0.85

Thus, the obtained analytical solution of the problem can be used in optimization of the thermal treatment process with respect to criteria of minimum energy consumption and the best quality of flat items with organic thermoreactive coatings.

NOTATION

τ, time; δ_1, coating thickness; δ_2, item thickness; T, temperature; λ, thermal conductivity; a, thermal diffusivity; α, coefficient of convective heat transfer; ε, effective blackness of coating; σ_0, Stefan-Boltzmann constant; k, constant coefficient; U, activation energy; R, gas constant; $T_1(\tau)$, thickness-averaged coating temperature.

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